International Journal of Education
Vol. 13 No. 2, Dec-2020, pp. 105-112

doi: 10.17509/ije.v13i2.23276

MATHEMATICAL PROBLEM-SOLVING BEHAVIORS OF THE ROUTINE SOLVER

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First draft received: 14 Feb 2020 Date Accepted: 1 Dec 2020 Final proof received: 23 Dec 2020

Abstract
The research refined the theories of students' characteristics in solving mathematical problems and constructed a holistic understanding of their problem-solving behaviors. It aimed at describing the problem-solving behaviors of a routine problem solver. The routine bridged the gap between the expert and the novice. This research used a qualitative approach which was carried out in six stages. The research participant was Rina (female, pseudonym), one of the 11th-grade students from one of the high schools in Palangka Raya City, Central Kalimantan, Indonesia. The selection of the participant was based on certain criteria. The instruments were three mathematics problems and semi-structured interviews. The trustworthiness of the research was fulfilled through credibility, dependability, and transferability checking. The results showed that the routine could understand problems and was able to identify the known and the target. The routine only focused on developing a plan which was based on a lack of concepts, limited previous experiences, or limited strategies. Problem-solving behaviors of the routine were between the expert and the novice.
Keywords: cognitive processes; mathematical problems; problem-solving behaviors; the routine solver

To cite this paper (in APA style):
Mairing, J. P. (2020). Mathematical problem-solving behaviors of the routine solver. *International Journal of Education, 13*(2), 105-112. DOI: 10.17509/ije.v13i2.23276

INTRODUCTION

Problem-solving is an integral part of and playing important role in mathematics learning. Mathematics curricula of the 21st century address the ability to solve problems as the main goal at all education levels. There are two important roles of problem solving in mathematics classes. Firstly, students can develop HOTS (higher order thinking skills) by learning to solve mathematics problems. Secondly, students acquire positive attitudes, which are habits of persistence, curiosity, and confidence in unfamiliar situations by trying to solve problems (National Council of Teachers of Mathematics, 2000; Reiss & Torner, 2007).

The mathematics problem is different from a routine question. The problem is a challenging and unfamiliar situation in which the part of the solution is not immediately visible to students (Mairing, 2018). Students cannot find the answer to the problems by applying certain formulas or procedures directly. In other words, the solution is a series of non-algorithmic steps(Reiss & Torner, 2007). On the other hand, students can determine the answer to the routine question by applying some formulas or procedures directly. The solution path can be seen by the students immediately (Posamenteir & Krulik, 2009). Therefore, the answer to a routine question is algorithmic steps.

The students' ability to solve problems could be classified as novice and expert problem solvers (Sternberg & Sternberg, 2012). The differences between the two solvers are characterized by schema, organization, the use of time, representation of the problems, work direction, strategy, automatization, efficiency, prediction of the difficulty, monitoring, accuracy of the solution, unusual problems confronting, and contradictory information handling. The experts need more time to understand and to represent the problems than to develop solution plans, compare to what the novices do. The experts also spend more time in defining problems and activating relevant prior knowledge than what the novices do (Goldstein, 2011; Gruwel et al., 2015). In the work direction, the experts use the means-end heuristic of forwarding, while the novices use a backward heuristic (Sternberg & Sternberg, 2012). The means-end is a problem-solving heuristic to reduce the difference between the known, and the target of problems (Goldstein, 2011). If students move from the known to the target, they used the forward heuristic. Conversely, they use the backward heuristic. The differences also occur in scanning, processing, organizing, and presenting information (Gruwel et al., 2015). They are significantly influenced by mathematics knowledge, general intelligence, general creativity, or the verbal ability of the solvers (Bahar & Maker, 2015).

Some research specifically described the problem-solving behaviors of the experts. The experts used a four-stage multidimensional problem-solving framework to solve the problems, namely orientation, planning, executing, and checking. The framework contained two cycles in which each cycle consisted of at least three of the stages. The planning phase contained sub-cycles namely conjecture, imagination, and evaluation (Carlson & Bloom, 2005). Mairing (2011) conducted research aimed at describing the thinking process of two experts who were medalists of the Mathematics National Olympiad. The research result was described in four problem-solving stages of Polya which were understanding problems, developing plans, carrying out the plans, and looking back (Polya, 1973).

The problem-solving behaviors of the novices have been described by Mairing (2017). The novices did not understand the problem. They only copied information from the problems without constructing the mental images. Their solution plans were based on the remembered formulas, even though the knowledge of the formulas was limited so the novices could not find the correct answers to the problems.

Furthermore, there were gaps between the experts and the novice that the experts had certain behaviors that the novices did not. At the stage of understanding the problems, the expert could construct mental images of the problems, while the novices could not identify important information about the problems, so they failed to construct the images. At the stage of developing plans, the experts' plans were based on the meaningful problem-solving schema, while the novices' plans were based on the limited formulas or the limited strategies. Therefore, the experts could see the solution paths, but the novices could not see them. At the stage of carrying out the plans, the experts carried out the plans systematically so they could find the correct answers, while the novices did some mistakes in implementing the plans so they could not find the answers. At the stage of looking back, the experts checked the answers by substituting them to certain equations, or check the solutions rows, while the novices checked the formulas or the calculation only because of their limited understanding and plans (Mairing, 2011, 2012, 2017).

The other researchers classified problem solvers into three groups, namely the novices, the routines, and the experts (Muir et al., 2008). Such classifications should refine the theories of students' behaviors in solving the problems, although the gaps between the experts and the novices raised some questions. Were the routine's behaviors between the expert and the novice? What were the behaviors of the routine when understanding problems, developing plans, carrying out the plans, and looking back? Did the routine able to identify the known and the target of the problems? Could the routine construct the appropriate mental images? Did the plans help the routine to see the answers? How did the routine carry out the plans? Did the routine look back on the solutions? If the routine did, how did the solver do it? If the routine did not, why did the solver perform it?

Therefore, this research was aimed at describing problem-solving behaviors of the routine. The theoretical framework of this research was Polya's stages. The stages were specifically used to solve mathematical problems. Also, the other problem-solving stages (Goldstein, 2011; Pape, 2004; Sternberg & Sternberg, 2012) corresponding to Polya's stages. The results of this research could answer those questions and complement the previous research to construct a holistic understanding of students' problem-solving behaviors. This understanding can help cognitive psychologists or mathematics teachers to develop theories, teaching methods, or learning plans intended to improve students' ability to solve mathematics problems.

METHOD

Design of the Research

This qualitative research was conducted under an interpretative study paradigm. The main characteristic of the research was describing and interpreting a process from students' points of view, namely behaviors of the routine as solving mathematics problems. Furthermore, the research also identified recurrent patterns, namely the solution path of the routine based on the behaviors (Ary et al., 2006).

Materials

The instruments were three mathematics problems and semi-structured interview guidelines. These problems were related to polynomials concepts. The problems were:

1. If polynomial $f(x)$ is divided by $x+1$ and $x-3$, the remainings are -2 and 7 respectively. If polynomial $g(x)$ is divided by $x+1$ and $x-3$, the remainings are 3 and 2 respectively. Let $h(x) = f(x).g(x)$. If $h(x)$ is divided by x^2-2x-3, the remaining is ...
2. If polynomial $p(x)$ is divided by x^2-x and x^2+x, the remainings were $4x+2$ and $4x+2$ respectively. If $p(x)$ is divided by x^2-1, the remaining is ...
3. If $(x-1)^2$ divide ax^4+bx^3+1, then $ab =$...

The questions in the guideline can be seen in Table 1.

doi: 10.17509/ije.v13i2.23276

Table 1
The interview guideline

The stages	The questions
Understanding the problem (*The researcher asked the participant after reading the problem*)	1. How many times have you read the problem? Why? 2. What is the meaning of the writings/images/symbols? 3. Can you retell this problem? 4. What is the known? 5. What is the unknown or the target?
Developing the plan (*The researcher asked the participant before writing the solution*)	1. What do you do first to solve the problem? Then? (*and so on*) 2. Do you have another plan? Please, explain.
Carrying out the problem (*The researcher asked the participant after writing the solution*)	1. Please, justify your writings. 2. Are you having difficulties when solving the problem? Please, explain. What is your idea to solve the difficulties? 3. Is the solution following the plan? 4. Do you check the solution steps? How to do it?
Looking Back (*The researcher asked the participant after justifying the solution*)	1. Are you sure the answer is correct? 2. Is there another way to solve this problem? What is your idea?

Participant

The researcher determined a research participant. The criterion was a student who had a fair ability to solve problems and was able to have both oral and written communication. The data of the ability was obtained by giving two problems, which differed from the problems posed to all students of a mathematics and science class of the 11th grade from one of the state high schools in Palangka Raya, Central Kalimantan, Indonesia. The result showed that the number of the experts, the routines, and the novices were 1 (2.5%), 9 (22.5%), and 30 (75%) respectively. Besides, the selection of the participant was also based on the suggestions given by the mathematics teacher regarding the student's skills in both oral and written communication. The participant was Rina (female), one of the nine students having a fair ability. The researcher asked consent from her parents to do in-depth interviews at her house.

Procedures

The research was carried out in six stages. First, the researcher determined the focus of the research which was the problem-solving behaviors of the participant at each Polya's stage. Second, the researcher developed instruments to collect data, namely three mathematics problems, and semi-structured interview guidelines. Third, the researcher chose the research participant that satisfied the participant's criteria. Fourth, the researcher collected data by conducting in-depth interviews based on the problems. Fifth, the researcher analyzed the data. The last, the researcher interpreted and verified the results of analyzing data (Ary et al., 2006).

Data Collection

The data collection was done by conducting in-depth interviews with the participant based on the problems. The interviews were conducted in the participant's house. The researcher came to her home for four months to conduct the interviews. The interviews were recorded using audio-visual recorders and carried out based on Polya's stages. Firstly, the participant read the problems. She could write or draw her understanding on paper but had not been allowed to write the solutions. The reading activities stopped until the participant said "already" or "finished". The researcher asked some questions to explore her understanding of the problems. Secondly, the participant communicated her plans to solve the problems. Thirdly, the participant wrote an implementation of the plans until she said "already" or "finished". The researcher asked her to explain each row of her solutions. Fourthly, the researcher explored the means of the participant to look back on the solutions by asking some questions.

Data Analysis

The data analysis was implemented in three steps. Firstly, the researcher transcribed the interview recordings and reduced the data of transcripts by giving some codes. The codes consisted of five digits. The first and the second digits stated the problem number, for example, code M1 stated the line of the transcript from the first problem. There were three possibilities of the code, namely M1, M2, or M3. The third stated that the problem-solving stage, namely U, P, C, or L stated **U**nderstanding, **P**lanning, **C**arrying out the plan, or **L**ooking back respectively. The fifth and sixth codes stated the order of activities in each stage. For example, M2C12 stated that the line of the transcript from the second problem, the participant did the twelfth activity of carrying out the planning stage.

Secondly, the researcher interpreted the transcripts and codes. The interpretation was conducted by giving some meaning and explanation of the data. The method was to analyze words/phrases/sentences and constant comparison. The analysis was conducted by reading the interview transcripts, focusing on words/phrases/sentences that were significantly interesting, listing possible meanings of them that appeared in the researcher's mind, and returned to the transcripts to determine the appropriate meanings (Strauss & Corbin, 1998). The comparison was done by comparing a certain category with others so the researcher found some behaviors that had the same characteristics.

Thirdly, the researcher and the participant verified the interpretation by evaluating criteria for credibility, dependability, and transferability. The credibility of this research was satisfied by prolonged engagement with the participant for four months, persistent/consistent observations, triangulation, structural relationships, and member checks. The researcher sent the transcripts and summaries of the researcher's conclusions to the participant for review. Triangulation was done by examining the solution written by the participant, and the interview transcripts (the triangulation method), or checking the participant's solution of a problem against solutions of other problems (the time triangulation). The dependability was satisfied by making clear and detailed documentation of collecting and analyzing data (leaving an audit trail). The transferability was satisfied by providing a complete description of the research participant and the context of the research took place (Bloomberg & Volpe, 2008; Lodico et al., 2006).

FINDINGS AND DISCUSSION

The interviews were conducted for four months in the participant's house. The results of the interviews were transcribed and coded. An activity could be the problem-solving behavior of the participant if it appeared in all problems to be shown by the second digit code of the activity which was 1, 2, and 3. For example, the participant was able to explain the known and the target of problems. This activity got code M1U02–03; M2U01–05, M3U04-09, 11, 12 so it appeared as the participant solved the first, second, and third problems. The researcher used three problems because the participant could not determine the answer to the first problem. The participant created the incorrect answer to the second and the third problems. However, the participant performed the same behavior to solve all problems. In other words, the activity had coded M1xxx, M2xxx, and M3xxx. The problem-solving behavior from the routine based on Polya's stages is as follows.

The Stage of Understanding the Problems

The participant was Rina (female). Rina read each problem 4 times for 3-7 minutes. She read repeatedly to understand the problems, and to develop the solution plans. Rina said, "because I am confused about what to do first". The participant could determine and explain the known and the target of the problems (M1U03–04, M2U05, M3U07–09). The explanation was not based only on the sentences in the problems. She used the previous schema in her mind. The following are the interview transcripts.

Researcher	:	What is the unknown?	(Codes)
Rina	:	The unknown is the remaining of $h(x)$ is divided by $x^2 - 2x - 3$	M1U03
Researcher	:	What is $h(x)$ itself?	
Rina	:	$h(x)$ itself is the product of $f(x)$ and $g(x)$	M1U04
Researcher	:	Okay, what is the unknown?	
Rina	:	The unknown is the remaining of $p(x)$ if it is divided by $x^2 - 1$	M2U05
Researcher	:	Okay, what is the unknown?	
Rina	:	The unknown is a times b	M3U07
Researcher	:	What is a?	
Rina	:	a is the coefficient of x^4	M3U08
Researcher	:	Then, what is b?	
Rina	:	b is the coefficient of x^3	M3U09

The Stage of Developing the Plans

The plans were developed by the participant as she read the problems. She also read more than once to check the plans. She did the checking because of finding the difficulties to see the solution paths, and of finding correct answers. The participant was not sure about the plans, but she did not develop others (M1P18, M2P16, M3P13). She still focused to develop the initial plans. Thus, one of the problem-solving behaviors of the participant was to focus on developing some limited plans to solve the problems, although she could not see the answers by using them.

The limited plans were based on the lack of the concepts or limited solution strategies, namely substituting some values to the polynomial, stating the polynomial = divisor × result of division + remainder, eliminating the equations obtained, or Horner Division method. The participant could not see the correct answers using the plans, and found difficulties to determine them. Therefore, she seemed hesitant and needed to read the problems repeatedly at least three times when explaining the plans (M1P12, M2P14, M3P11). The participant could not solve the difficulties. She planned the next steps while writing the solutions. The following are the interview transcripts.

Researcher	:	Then ...	(Codes)
Rina	:	Eh ... (thinking while reading the problem) Eh ... (thinking while reading the problem), then ... (thinking while reading the problem)	M1P09
Researcher	:	Do you want to explain the plan again or to try to solve the problem?	
Rina	:	I want to try to solve ...	M1P10
Researcher	:	Before writing the solution, the plan stops here, what are the obstacles?	
Rina	:	The obstacle is ... eh ...	M1P11
Researcher	:	Okay, the plan stops here. Can the plan find the	

		unknown?	
Rina	:	Not yet	M1P12
Researcher	:	Is there another plan to solve this problem?	
Rina	:	Not yet	M1P18
Researcher	:	Could you find the answer by using the plan?	
Rina	:	Not yet	M2P12
Researcher	:	It means the steps have not finished yet, so after that?	
Rina	:	After that, eliminating between divisor eh ... between ... (becoming silent, her hands touch her mouth while looking at the problem at 15:00 to 15:53) between the first and the second divisors ...	M2P13
Researcher	:	After eliminating, do you find the answer?	
Rina	:	Not yet ...	M2P14
Researcher	:	Are there the next steps?	
Rina	:	(becoming silent while leaning chin with her hand from 16:00 to 17:04)	M2P15
Researcher	:	Previously, Rina explained the plan but you can not find the answer, is there another plan to solve this problem?	
Rina	:	Not yet	M2P16
Reseacher	:	Ok, after dividing by $x-1$, can you get a times b [the target]	
Rina	:	Not yet	M3P09
Researcher	:	It means there are still some further steps, can you explain further?	
Rina	:	Eh ... by ... elimination	M3P10
Researcher	:	What are the equations?	
Rina	:	Between ... uh ... between ... $p(x)$ and ... previously using the horner method, the one below is the result of division and the rest, meaning later eh ... (becoming silent and looking at the problem from 17:50 - 19:01) wanting to try writing ...	M3P11
Researcher	:	Is there another plan to solve it?	
Rina	:	Not yet	M3P13

The Stage of Carrying Out the Plans

The participant carried out each plan for 17-42 minutes. She focused to write the solutions according to the initial plans. Some parts of the solutions differed from the initial plans. The parts were planned while carrying out the initial plans. The participant could justify the solutions, but she did not realize that some parts of the solutions were wrong. *At the first problem*, the participant represented function formulas of f and g using the same variables namely a, b, and c, or stated the degree of function f was 2, she said: if the remaining degree is 0, then a degree of f can be 1" (Fig. 1), *At the second problem*, she reused previous rows, namely $p(1)=6$ and $p(-1)=-2$, even though the context was different. Similarly, she used different facts for the remaining. She said: "because of $x^2-1=(x+1)(x-1)$ so that the remainder of $p(1)=6$ and $p(-1)=-2$ were also multiplied, therefore the remainder asked for the problem was $6\times(-2)=-12$ (Fig.2), *At the third problem*, she could implement the plan in the first stage, namely dividing $p(x)$ by $x-1$using the Horner method. Then, the result was divided again by $x-1$ using the method. However, there was an error that was not realized by the participant initially. She wrote a, $b+a$, $b+a$, and $b+1$, whereas the last term should be $b+a$. The participant could write the result of the second division by the method, but the remaining was wrong because the previous was an error. The other error occurred at the solution line of $3a+3b+1$ that was obtained from the remaining of the second division. The participant made the remainder of $3a+3b+1$ was 0, but she could not explain the reasons She also made the remainder in part (a) of 0 (Fig.3). The activities became her habit of solving problems. The following are the interview transcripts.

Researcher	:	Okay, how do you get -1 here?	(Codes)
Rina	:	This is the remaining of $3a+3b+1$ equals to 0, then I move to the other side [of the equation]	M3C06
Researcher	:	But the remaining is not equal to 0, so why do you make it equal to 0?	
Rina	:	Because (while smiling) ... because ... (becoming silent) is equal to 0	M3C07

The participant could not explain the difficulties of carrying out the plans. Therefore, she was not sure about the solutions indicated by smiling or being silent when trying to explain them.

Figure 1
The Solution of the First Problem

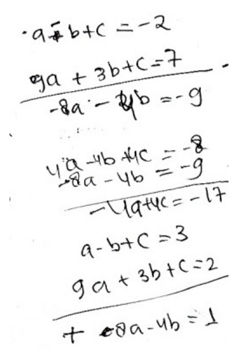


Figure 2

The Solution of the Second Problem

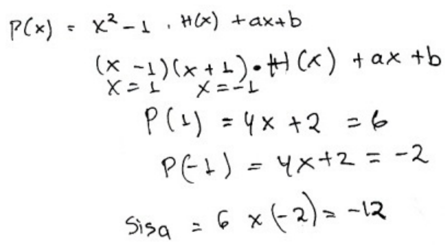
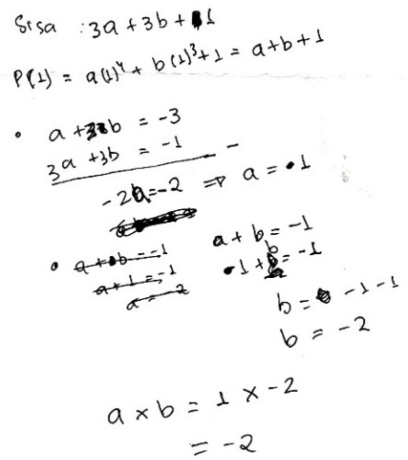


Figure 3
The Solution of the Third Problem



The Stage of Looking Back
The participant did not look back on the solutions because she could not find the correct answer (the first problem), or she realized that the answers were not correct but having no other plans to find the answers (the second and the third problems). The following are the transcripts of the second and the third problems.

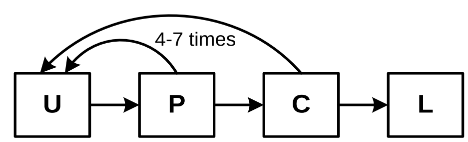
Researcher	:	Are you sure that the answer is correct?	(Codes)
Rina	:	No ... (smiling)	M2L01
Researcher	:	Have you checked the solution?	
Rina	:	No.	M2L02
Researcher	:	Okay, is it the correct answer?	
Rina	:	Actually, there is something wrong here (pointing to $b+1$ on the second Horner). It should be $b+a$, but written $b+1$	M3L08

The errors were not realized by the participant when carrying out the plans. It showed that she did not check the lines of the solutions.

The research results were in line with those of previous research. The routine solver read the problems repeatedly to understand the problem. The solver could determine the important information of the problems, namely the known and the target. The information was used to develop the limited plans, but the plans could not bridge the known and the target. The solver carried out the plans which led to wrong answers. The solver looked back at the solutions, but the solver could not develop some alternative plans to get the correct answers (Khasanah et al., 2018; Setiawani et al., 2019). Meanwhile, there was a difference between the results and that was found by Sanjayaet al. (2018). They stated that some parts of the information could not be identified by the solver at the step of understanding the problem.

Based on the results, the solution path of the participant can be seen in Fig.4. The path reinforced theories of the cyclic nature of problem-solving (Carlson & Bloom, 2005).

Figure 4
The routine's solution path



Note: U = understanding problem, P = developing plan, C = carrying out the plan, and L = looking back

The expert constructed the mental images to understand the problems and developed two plans for each problem. The novice could not understand the problems, and the solution plans were inappropriate (Goldstein, 2011; Gruwei et al. 2015; Mairing et al., 2011; Mairing, 2017; Sternberg & Sternberg, 2012). On the other hand, the routine could understand the problems, but devised limited plans. The plans are based on a lack of concepts, limited experiences, and limited strategies. Also, the expert could justify the solutions and checked them. The novice made errors in implementing the plans. The novice also could not justify the solutions, and the novice was not sure of the answer. Whereas the routine was able to justify the solutions, but the limited plans made the routine did not realize that some parts of the solutions were inappropriate. Therefore, the problem-solving behaviors of the routine were between the expert and the novice.

The different behaviors of the three solvers occurred because their cognitive processes were different. Problem-solving could be seen as a cognitive process that integrates information processing, comprehension, reasoning, analogical transfer, cognitive styles, and attitudes to solve problems(Botia & Orozo, 2009). The differences could be explained

doi: 10.17509/ije.v13i2.23276

using a cognitive model of the problem-solving, namely decoding, representing, processing, and implementing (Singer & Voica, 2013). At decoding activities, the expert was able to transform texts in the problems into relationships between the data and the meaningful concepts (Mairing et al., 2011). The novice failed to transform texts because the novice did not have concepts related to the problems (Mairing, 2017). The routine was able to transform texts using appropriate concepts, but they were isolated, so the devising plans used limited concepts.

The success of the transformation of the text was influenced by the reading behaviors of the solvers. Research showed that the routine and the novice had the reading behaviors of DTA (Direct Translation Approach). The main characteristic of the behaviors was the students' lack of evidence of the data transformation, relationships between the data, the problem context, and the related concepts. On the other hand, the expert had the reading behaviors of an MBA (Meaning-Based Approach). The main characteristic of an MBA was to record and to organize the data in appropriate contexts to construct mental models (Mairing et al., 2012; Pape, 2004).

At representing activities, the expert constructed appropriate mental models of problem conditions and related them to mathematics concepts and previous experiences to define and to represent the problems. In other words, the expert used analogical thinking (Pretz et al., 2003). Construction of the models could not be done by the novice because the novice did not have a problem-solving schema. Whereas the routine constructed the mental models, but the routine devised limited plans. The limited plans were caused by a limited problem-solving schema. The problem-solving schema itself was a link between knowledge, namely the mental models of problems, the previous experiences, the meaningful understanding of concepts, and the understanding of strategies or approaches of problem-solving (Cadez & Kolar, 2015; Mairing, 2018).

CONCLUSION

The problem-solving behaviors of the routine filled the gap between the expert and the novice. In understanding the problems, the routine read the problems many times. The purpose was to think of solution plans. The routine could understand problems, and able to explain the known and the target based on appropriate mathematics concepts related to the problems. At developing plans, the routine only focused on devising a limited plan for each problem. However, the plans were based on a lack of concepts, limited previous experiences, or limited strategies. The routine was not able to see the answers using the plans. Therefore, the routine planned the next steps while writing the solutions. At carrying out the plans, the routine focused on implementing the initial plans. The routine was able to reason the solutions. However, the limited plans made the routine unable to realize that some parts of the solutions were wrong. The condition made the routine not sure with the solutions or the answers. At a looking-back stage, the routine realized that the answers or the solutions were not correct so the routine did not look back at the solutions. The routine did not have other plans to find the correct answers.

More studies in various contexts need to be carried out to construct a holistic understanding of problem-solving behaviors of the routines. The understanding will help cognitive psychologists and teachers to develop learning methods to improve students' problem-solving ability. How do teachers improve the novices to become the routines? How do teachers increase the ability of the routines to become the experts?

ACKNOWLEDGMENTS

The researcher would like to thank the administrators of Palangka Raya University and SMAN 2 Palangka Raya for promoting and facilitating this research. The researcher also thanks the participant's parents for allowing some interviews, so this research could be completed.

REFERENCES

Ary, D., Jacobs, L. C., & Sorensen, C. (2006). *Introduction to research in education* (8th ed.). Wadsworth.

Bahar, A., & Maker, C. J. (2015). Cognitive backgrounds of problem solving: A comparison of open ended vs. closed mathematics problems. *Eurasia Journal of Mathematics, Science and Technology Education, 11*(6), 1531-1546. 10.12973/eurasia.2015.1410a.

Bloomberg, L. D., & Volpe, M. (2008). *Completing your qualitative dissertation: A roadmap from beginning to end.* Sage Publications, Inc.

Botia, M. L., & Orozo, L. H. (2009). Crtitical review of problem solvng process traditional theoritical models. *International Journal of Psychological Research, 2*(1), 62-72.

Cadez, H. T., & Kolar, M. D. (2015). Comparison of types of generalizatons and problem-solving schemas used to solve a mathematical problems. *Educational Studies in Mathematics, 89*, 283-306.

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: an emergent multidimensional problem solving framework. *Educational Studies in Mathematics, 58*, 45-75. 10.1007/s10649-005-0808-x.

Goldstein, E. B. (2011). *Cognitive psychology: Connecting mind, research, and everyday experience* (3th ed.). Wadsworth.

Gruwel, S. B., Wopereis, I., & Vermetten, Y. (2015). Information problem solving by experts and novices: Analysis of a complex cognitive skill. *Computer in Human Behaviour, 21*(3), 487-508. doi: 10.1016/j.chb.2004.10.005.

Khasanah, V. N., Usodo, B., & Subanti, S. (2018). Student's thinking process in solving word problems in geometry. *Journal of Physics: Conference Series*, 1-6. 10.1088/1742-6596/1013/1/012133.

Lodico, M. G., Spaulding, D. T., & Voegtle, K. H. (2006). *Method in educational research: From theory to practice.* John Willey & Sons, Inc.

Mairing, J. P. (2018). *Pemecahan masalah matematika: Cara siswa memperoleh jalan untuk berpikir kreatif dan sikap positif* [*Mathematics problem solving: How students acquire creative thinking and positive attitudes*]. Alfabeta.

Mairing, J. P. (2017). Thinking process of naive problem solvers to solve mathematical problems. *International Educational Studies, 10*(1), 1-10. 10.5539/ies.v10n1p1.

Mairing, J. P., Budayasa, I. K., & Juniati, D. (2012). Perbedaan profil pemecahan masalah peraih medali OSN matematika berdasarkan jenis kelamin. *Jurnal Ilmu Pendidikan, 18*(2), 125–134. 10.17977/jip.v18i2.3612.

Mairing, J. P., Budayasa, I. K., & Juniati, D. (2011). Profil pemecahan masalah peraih medali OSN. *Jurnal Pendidikan dan Pembelajaran, 18*(1), 65–71.

Muir, T., Beswick, K., & Williamson, J. (2008). I am not very good at solving problems: An exploration of student's problem solving behaviours. *The Journal of Mathematical Behaviour, 27*(3), 228–241.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* The National Council of Teachers of Mathematics, Inc.

Pape, S. J. (2004). Middle school children's problem solving behavior: A cognitive analysis from a reading comprehension perspective. *Journal for Research in Mathematics Education, 35*(3), 187-219. 10.2307/30034912.

Polya, G. (1973). *How to solve it* (2 ed.). Princeton University Press.

Posamenteir, A. S., & Krulik, S. (2009). *Problem solving in mathematics grades 3–6, powerful strategies to deepen understanding.* Corwin A SAGE Company.

Pretz, J. E., Naples, A. J., & Sternberg, R. J. (2003). Recognizing, defining, and representing problems. In T. P. Solving, *The psychology of problem solving* (pp. 3-30). Cambridge University Press.

Reiss, K., & Torner, G. (2007). Problem solving in the mathematics classroom: The German perspective. *ZDM Mathematics Education, 39*, 431-441.

Sanjaya, A., Johar, R., Ikhsan, M., & Khairi, L. (2018). Students' thinking process in solving mathematical problems based on the levels of mathematical ability. *Journal of Physics: Conference Series*, 1-7. 10.1088/1742-6596/1088/1/012116.

Setiawani, S., Fatahillah, A., Dafik, Oktavianingthyas, E., & Wardani, D. Y. (2019). The students' creative thinking process in solving mathematics problem based on wallas' stages. *IOP Conference Series: Earth and Environmental Science.243*, pp. 1-8. IOP Publishing Ltd. 10.1088/1755-1315/243/1/012052.

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem posing tasks. *Educational Studies in Mathematics, 83* (1), 9-18.

Sternberg, R. J., & Sternberg, K. (2012). *Cognitive psychology* (6 ed.). Wadsworth Cengage Learning.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research, tehniques and procedures for developing grounded theory.* SAGE Publications, Inc.